Strangeness -2 and -3 Baryons in a Constituent Quark Model*

Muslema Pervin[1] and W. Roberts[2]
[1] *Physics Division, Argonne National Laboratory, Argonne, IL 60439*
[2] *Department of Physics, Florida State University, Tallahassee, FL 32306*

We apply a quark model developed in earlier work to the spectrum of baryons with strangeness -2 and -3. The model describes a number of well-established baryons successfully, and application to cascade baryons allows the quantum numbers of some known states to be deduced.

PACS numbers: 12.39.-x, 12.39.Jh, 12.39.Pn, 14.20.Jn

JLAB-THY-07-728

I. INTRODUCTION AND MOTIVATION

The study of hyperon properties can provide important insight to two questions of crucial interest to hadronists. The first of these is 'what are the relevant degrees of freedom in a baryon', and is in some sense subsumed in the second, 'what is the mechanism of confinement?'. In order to understand the symmetries and dynamics of the strong interaction, the expected multiplet structure of the baryons must be established experimentally, and details of their excitation spectrum are crucial. However, there has not been much information available on hyperons, particularly those with $S < -1$, where S is the strangeness of the baryon. This means that neither the multiplets nor the excitation spectrum of the light baryons are well established.

The most recent version of the Particle Data Group (PDG) listings [1] notes: *Not much is known about Ξ resonances. This is because (1) they can only be produced as a part of a final state, and so the analysis is more complicated than if direct formation were possible, (2) the production cross sections are small (typically a few μb), and (3) the final states are topologically complicated and difficult to study with electronic techniques. Thus early information about Ξ resonances came entirely from bubble chamber experiments, where the numbers of events are small, and only in the 1980s did electronic experiments make any significant contributions. However, nothing of significance on Ξ resonances has been added since our 1988 edition.* Much of this comment is also valid for Ω baryons.

There are only three multistrange baryons with four star ratings whose spin and parity are known, as reported in the most recent PDG listings [1]. These are the ground states $\Xi(J^P = 1/2^+)$ and $\Omega(3/2^+)$, and one excited cascade, $\Xi(3/2^+)$, with masses 1317 MeV, 1672 MeV and 1534 MeV, respectively. However, the parity of the lowest lying Ξ has not been determined experimentally, but positive parity is expected. The spin and parity of the Ω^- have only been experimentally determined recently [2]: the assignment of $J^P = 3/2^+$ was based on assignment of the state to the baryon decuplet. The PDG notes that for the $\Xi(3/2^+)$ at 1534 MeV, "[s]pin-parity $3/2^+$ is favored by the data". There are nine other excited cascades and three other Ω's reported by PDG, but these all have three-star or lower ratings, with few of their quantum numbers determined. Among the three-star states, the $\Xi(1823)$ achieves a J^P assignment of $3/2^-$ by virtue of one experiment that favors $J = 3/2$ but which cannot make a parity assignment [3], and one experiment that determines that J is consistent with 3/2 and which favors negative parity [4]. Among the other three-star states, it is known that the state at 2030 MeV has $J \geq 5/2$ [5], but the quantum numbers of no other states have been ascertained. Among the Ω baryons, the only state that has quantum number assignments is the ground state at 1672 MeV. The status of the properties of these multistrange baryons is summarized in Table I.

Recently, after a hiatus of nearly twenty years, there has been increasing experimental interest in hyperons with $S < -1$, due in part to the large samples of beauty and charmed hadrons produced at Cleo and the B

* Notice: Authored by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177. The U.S. Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce this manuscript for U.S. Government purposes.

TABLE I: The Ξ and Ω baryons as listed in the most recent Particle Data Group Listings [1]

Experimental State	J^P	PDG rating	Experimental State	J^P	PDG rating
$\Xi(1317)$	$1/2^+$ (expected)	****	$\Omega(1672)$	$3/2^+$ ([2])	****
$\Xi(1530)$	$3/2^+$ (favored by data)	****	$\Omega(2250)$	$?^?$	***
$\Xi(1823)$	$3/2^-$ ([3, 4])	***	$\Omega(2380)$	$?^?$	**
$\Xi(1690)$	$?^?$	***	$\Omega(2470)$	$?^?$	**
$\Xi(1950)$	$?^?$	***			
$\Xi(2030)$	$\geq 5/2^?$ ([5])	***			
$\Xi(2250)$	$?^?$	**			
$\Xi(2370)$	$?^?$	**			
$\Xi(1620)$	$?^?$	*			
$\Xi(2120)$	$?^?$	*			
$\Xi(2500)$	$?^?$	*			

factories, and the large number of multi-particle decays made accessible by these samples. Results from BaBar [2, 6, 7], preliminary results from Jefferson Lab (JLab) [8, 9], and plans for experiments at JLab suggest that more high-precision data on these states will be forthcoming in the not-too-distant future. In addition, a number of activities related to these hyperons are being carried out elsewhere around the world. For example, there have been measurements of weak decays of cascades (Ξ^0) by the KTeV Collaboration [10], as well as by the NA48/I Collaboration [11], in addition to studies of cascade resonances in relativistic heavy-ion collisions [12].

The BaBar Collaboration at SLAC has been pursuing studies to measure the masses, widths, spins and parities of a number of excited hyperons, including the $\Xi(1690)$. The PDG gives this state a three-star rating, but its quantum numbers are undetermined. In addition, the BaBar Collaboration has examined the $\Xi(1530)$ and $\Omega(1672)$ to determine their quantum numbers. The result of these analyses is that a spin of 1/2 for the $\Xi(1690)$ is better supported by experiment than spin 3/2 or 5/2, but the parity remains undetermined. For the $\Xi(1530)$, the J^P is determined as $3/2^+$, for the first time. For the $\Omega(1672)$, the BaBar Collaboration concludes that the spin is consistent with 3/2, if the decaying baryon has spin 1/2 (the processes studied are $\Omega_c^0 \to \Omega^- K^+$ and $\Xi_c^0 \to \Omega^- \pi^+$) [2, 6].

On the theoretical side, there have been a few treatments of baryons with $S = -2$ and $S = -3$. Within the framework of the constituent quark model, Chao, Isgur and Karl [13] used a nonrelativistic quark model, while Capstick and Isgur [14] used a relativized version, both of which were based on one-gluon-exchange. Glozman and Riska [15, 16] used a one-boson-exchange model to look at these states, while Oh [17] examined the hyperon spectrum in a Skyrme model. QCD sum rules [18] have also been used to examine these states, as has the collective excitation model of Bijker, Iachello and Leviathan [19]. A number of authors have also examined these states in the framework of the large N_c expansion [20].

All of these treatments describe the ground states of the Ξ and the Ω spectrum successfully, but provide a range of predictions for the masses of the excited states. For the lowest lying $\Xi(1/2^-)$, for instance, predicted masses range from 1550 MeV to 1869 MeV, with all but one of the approaches predicting masses larger than 1750 MeV. A spread of 100 to 200 MeV in the mass of any particular state is not uncommon among the predictions of these various treatments, particularly for excited states.

In the work we present in this manuscript, we use a nonrelativistic quark model to obtain the excitation spectrum for multistrange baryons. In our calculation we fit a number of the experimentally well-known baryons to fix the parameters of the model Hamiltonian. We then use the same Hamiltonian to predict masses (and spin-parity assignments) of a number of excited cascades and Omegas. As discussed above, an outstanding question in the hyperon sector is that of which states belong to which SU(6) multiplets, and this can only be determined with any certitude if the spins and parities of the states are known. The primary goal of this work is therefore to explore the excitation spectrum of states with $S < -1$, and in doing so perhaps provide some guidance to future experimental efforts that will examine these states. The model we use is one that we have developed for examining semileptonic decays of baryons [21, 22], and some details of the model are presented in the next section. Section III presents our results, and section IV of this manuscript provides our conclusions and describes possible future directions.

II. THE MODEL

Our starting point is a nonrelativistic quark model Hamiltonian, similar to that used by Isgur and Karl [23], and described in our earlier work [21, 22].

A. Hamiltonian

The phenomenological Hamiltonian we use takes the form

$$H = \sum_i K_i + \sum_{i<j} \left(V_{\rm conf}^{ij} + H_{\rm hyp}^{ij} \right) + V_{\rm SO} + C_{qqq}. \tag{1}$$

K_i is the kinetic energy of the ith quark, and takes the form

$$K_i = \left(m_i + \frac{p_i^2}{2m_i} \right). \tag{2}$$

The spin independent confining potential consists of linear and Coulomb components,

$$V_{\rm conf}^{ij} = \sum_{i<j=1}^{3} \left(\frac{b r_{ij}}{2} - \frac{2\alpha_{\rm Coul}}{3 r_{ij}} \right). \tag{3}$$

The spin-dependent part of the potential is written as

$$H_{\rm hyp}^{ij} = \sum_{i<j=1}^{3} \left[\frac{2\alpha_{\rm con}}{3m_i m_j} \frac{8\pi}{3} \mathbf{S}_i \cdot \mathbf{S}_j \delta^3(\mathbf{r}_{ij}) + \frac{2\alpha_{\rm ten}}{3m_i m_j} \frac{1}{r_{ij}^3} \left(\frac{3\mathbf{S}_i \cdot \mathbf{r}_{ij} \mathbf{S}_j \cdot \mathbf{r}_{ij}}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j \right) \right], \tag{4}$$

which consists of the contact and the tensor terms, with $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$. In this work we use a simplified spin-orbit potential that takes the form,

$$V_{\rm SO} = \frac{\alpha_{\rm SO}}{\rho^2 + \lambda^2} \frac{\mathbf{L} \cdot \mathbf{S}}{(m_1 + m_2 + m_3)^2}. \tag{5}$$

In this expression, L is the total orbital angular momentum and S is the total spin of the baryon. We note that this form is not very sensitive to the internal structure of the baryon.

B. Baryon Wave Function

In our model, a baryon wave function is described in terms of a totally antisymmetric color wave function, multiplying a symmetric combination of flavor, space and spin wave functions. The symmetric flavor-spin-space part of the baryon wave function is written as $\Psi_A^S = \phi_A(\text{flavor})\psi_A(\rho, \lambda)\chi_A(\text{spin})$, where $\rho = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2)$, $\lambda = \frac{1}{\sqrt{6}}(\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)$ are the Jacobi coordinates.

The total spin of the three spin-1/2 quarks can be either 3/2 or 1/2. The spin wave functions for the maximally stretched state in each case are

$$\begin{aligned} \chi_{3/2}^{S}(+3/2) &= |\uparrow\uparrow\uparrow\rangle, \\ \chi_{1/2}^{\rho}(+1/2) &= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle), \\ \chi_{1/2}^{\lambda}(+1/2) &= -\frac{1}{\sqrt{6}}(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle), \end{aligned} \tag{6}$$

where S labels the state as totally symmetric, while λ/ρ denotes the mixed symmetric states that are symmetric/antisymmetric under the exchange of quarks 1 and 2. In our model, we treat states with three identical quarks, such as the N, Δ and Ω, differently from states with one distinguishable quark, such as the Λ_Q, Σ_Q and Ξ.

1. States with $m_{q_1} = m_{q_2} \neq m_{q_3}$

For baryons containing one constituent quark having a mass different from that of the other two quarks, we symmetrize or antisymmetrize the flavor-spin-space part of the wave function only with respect to interchange of the two identical quarks, denoted 1 and 2 in our model. The flavor part of the wave function for such states can be either symmetric or antisymmetric in quarks 1 and 2. For Λ_Q-type baryons the flavor wave function is

$$\phi_{\Lambda_Q} = \frac{1}{\sqrt{2}}(ud - du)Q, \tag{7}$$

which is antisymmetric in quarks 1 and 2. The space-spin portion of such wave functions must therefore be antisymmetric in quarks 1 and 2, in order to yield a wave function that is symmetric in quarks 1 and 2. The flavor wave function of a Σ_Q-type baryon is

$$\phi_{\Sigma_Q} = \frac{1}{\sqrt{2}}(ud + du)Q, \tag{8}$$

which is symmetric in quarks 1 and 2. The space-spin part of the wave function must therefore be symmetric in quarks 1 and 2 to give the correct overall symmetry.

In either case, the spatial wave function for total $\mathbf{L} = \ell_\rho + \ell_\lambda$ is constructed from a Clebsch-Gordan sum of the products of functions of the two Jacobi coordinates ρ and λ, and takes the form

$$\psi_{LMn_\rho\ell_\rho n_\lambda\ell_\lambda}(\rho,\lambda) = \sum_m \langle LM|\ell_\rho m, \ell_\lambda M-m\rangle \psi_{n_\rho\ell_\rho m}(\rho)\psi_{n_\lambda\ell_\lambda M-m}(\lambda). \tag{9}$$

The spatial and spin wave functions are then coupled to give wave functions corresponding to total spin J and parity $(-1)^{(l_\rho+l_\lambda)}$. Thus,

$$\begin{aligned} \Psi_{JM} &= \sum_{M_L} \langle JM|LM_L, SM-M_L\rangle \psi_{LM_L n_\rho\ell_\rho n_\lambda\ell_\lambda}(\rho,\lambda)\chi_S(M-M_L) \\ &\equiv \left[\psi_{LM_L n_\rho\ell_\rho n_\lambda\ell_\lambda}(\rho,\lambda)\chi_S(M-M_L)\right]_{J,M}. \end{aligned} \tag{10}$$

The full wave function for a state A is then built from a linear superposition of such components as

$$\Psi_{A,J^P M} = \phi_A \sum_i \eta_i^A \Psi^i_{JM}. \tag{11}$$

In the above, ϕ_A is the flavor wave function of the state A, and the expansion coefficients η_i^A are determined by diagonalizing the Hamiltonian shown previously in the basis of the Ψ_{JM}. For this calculation, we limit the expansion in the last equation to components that satisfy $N \leq 2$, where $N = 2(n_\rho + n_\lambda) + \ell_\rho + \ell_\lambda$. For example, wave functions for a Ξ with $J^P = 1/2^+$ have the form

$$\begin{aligned} \Psi^{\Xi}_{1/2^+M} &= \phi_\Xi \Bigg(\left[\eta_1^\Xi \psi_{000000}(\rho,\lambda) + \eta_2^\Xi \psi_{001000}(\rho,\lambda) + \eta_3^\Xi \psi_{000010}(\rho,\lambda)\right] \chi^\lambda_{1/2}(M) \\ &+ \eta_4^\Xi \psi_{000101}(\rho,\lambda)\chi^\rho_{1/2}(M) + \eta_5^\Xi \left[\psi_{1M_L0101}(\rho,\lambda)\chi^\rho_{1/2}(M-M_L)\right]_{1/2,M} \\ &+ \eta_6^\Xi \left[\psi_{2M_L0200}(\rho,\lambda)\chi^S_{3/2}(M-M_L)\right]_{1/2,M} + \eta_7^\Xi \left[\psi_{2M_L0002}(\rho,\lambda)\chi^S_{3/2}(M-M_L)\right]_{1/2,M} \Bigg). \end{aligned} \tag{12}$$

Diagonalization of the Hamiltonian yields seven states having $J^P = 1/2^+$ composed of the components above, with the set of $\{\eta_i^\Xi\}$ being different for each state.

2. States with $m_{q_1} = m_{q_2} = m_{q_3}$

Baryons containing three identical quarks have the full $SU(6) \equiv [SU(3)_{\rm flavor} \times SU(2)_{\rm spin}]$ symmetry. In our spectrum calculation we use the SU(6) symmetric wave functions, see for example Ref. [23], for such states. For completeness we give a very brief description of these SU(6) wave functions. Up to the $N = 2$ harmonic oscillator level, there are five $SU(6)$ multiplets for positive parity baryons. In terms of the orbital quantum number L these multiplets are the 56 with $L = 0$ and $L = 2$, the 70, also with $L = 0$ and $L = 2$ and a 20 with $L = 1$. As an example, the predominant component of the ground state nucleon wave function is expected to be

$$\left|{}^2 8(56, 0^+)\frac{1}{2}^+\right\rangle = \frac{1}{\sqrt{2}}\left(\chi^\rho\phi^\rho + \chi^\lambda\phi^\lambda\right)\psi_{000000}(\rho,\lambda), \tag{13}$$

where $\psi_{000000}(\rho,\lambda)$ is the ground state ($l_\rho = l_\lambda = 0$) spatial wave function. The notation above is $\left|{}^{(2S+1)}\nu(\mu, L^P)J^P\right\rangle$, where S is the total spin of the quarks ($= 1/2$ or $3/2$), ν is the $\mathrm{SU}(3)_{\rm flavor}$ multiplet to which the state belongs, μ is its $SU(6) \equiv [SU(3)_{\rm flavor} \times SU(2)_{\rm spin}]$ multiplet, L is the total orbital angular momentum in the state, J is the total angular momentum, and P is the parity. For the proton,

$$\phi^\rho = \frac{1}{\sqrt{2}}\left(ud - du\right)u, \quad \phi^\lambda = -\frac{1}{\sqrt{6}}\left[\left(ud + du\right)u - 2uud\right]. \tag{14}$$

As with the states containing two different flavors of quarks, the wave functions of the states with three identical quarks are constructed from a linear superposition of all wave functions (up to $N = 2$) having the appropriate quantum numbers. Thus, for example, the wave functions for nucleons with $J^P = 1/2^+$ are written

$$\begin{aligned}\Psi^N_{1/2^+M} &= \left(\eta^N_1\left|{}^2 8(56,0^+)\frac{1}{2}^+\right\rangle + \eta^N_2\left|{}^2 8(56',0^+)\frac{1}{2}^+\right\rangle + \eta^N_3\left|{}^2 8(70,0^+)\frac{1}{2}^+\right\rangle\right.\\ &+ \left.\eta^N_4\left|{}^2 8(70,2^+)\frac{1}{2}^+\right\rangle + \eta^N_5\left|{}^2 8(20,1^+)\frac{1}{2}^+\right\rangle\right), \end{aligned}\tag{15}$$

with the set of $\{\eta^N_i\}$ being different for each state. The explicit forms for these SU(6)×O(3) wave functions in terms of the flavor, space and spin wave functions that we use are

$$\begin{aligned}\left|{}^2 8(56',0^+)\frac{1}{2}^+\right\rangle &= \frac{1}{2}\left(\chi^\rho\phi^\rho + \chi^\lambda\phi^\lambda\right)\left(\psi_{001000}(\rho,\lambda) + \psi_{000010}(\rho,\lambda)\right),\\ \left|{}^2 8(70,0^+)\frac{1}{2}^+\right\rangle &= \frac{1}{2\sqrt{2}}\left(\chi^\rho\phi^\rho - \chi^\lambda\phi^\lambda\right)\left(\psi_{001000}(\rho,\lambda) - \psi_{000010}(\rho,\lambda)\right) + \frac{1}{2}\left(\chi^\rho\phi^\lambda + \chi^\lambda\phi^\rho\right)\psi_{000101}(\rho,\lambda),\\ \left|{}^2 8(70,2^+)\frac{1}{2}^+\right\rangle &= \frac{1}{\sqrt{2}}\left[\phi^\rho\psi_{2M_L0101}(\rho,\lambda) + \frac{1}{\sqrt{2}}\phi^\lambda\left(\psi_{2M_L0002}(\rho,\lambda) - \psi_{2M_L0200}(\rho,\lambda)\right)\right]\chi^S,\\ \left|{}^2 8(20,1^+)\frac{1}{2}^+\right\rangle &= \frac{1}{\sqrt{2}}\left(\chi^\rho\phi^\lambda - \chi^\lambda\phi^\rho\right)\psi_{1M_L0101}(\rho,\lambda).\end{aligned}\tag{16}$$

For states with three identical quarks, as well as for states with only two identical quarks, we construct our wave functions using the harmonic oscillator basis. Each basis wave function takes the well-known form

$$\psi_{nLm}(\mathbf{r}) = \left[\frac{2\,n!}{(n+L+\frac{1}{2})!}\right]^{\frac{1}{2}}\alpha^{L+\frac{3}{2}}e^{-\frac{\alpha^2 r^2}{2}}L_n^{L+\frac{1}{2}}(\alpha^2 r^2)\mathcal{Y}_{Lm}(\mathbf{r}), \tag{17}$$

where $\mathcal{Y}_{Lm}(\mathbf{r})$ is a solid harmonic, and $L^\beta_n(x)$ is a generalized Laguerre polynomial. The size parameters α_ρ and α_λ appearing in the wave functions are treated as independent variational parameters. However, when the three quarks in the baryon are identical, the variational procedure automatically chooses $\alpha_\rho = \alpha_\lambda$.

III. NUMERICAL RESULTS

A. Hamiltonian Parameters and Baryon Spectrum

In the previous section, we introduced the Hamiltonian we use to obtain the baryon spectrum. There are ten free parameters to be determined for the baryon spectrum: four quark masses ($m_u = m_d$, m_s, m_c and m_b), and six parameters of the potential ($\alpha_{\rm con}$, $\alpha_{\rm tens}$, $\alpha_{\rm Coul}$, $\alpha_{\rm SO}$, b and C_{qqq}), and these are determined from a 'variational diagonalization' of the Hamiltonian. The variational parameters are the wave function size parameters α_ρ and α_λ of Eq. (17). This variational diagonalization is accompanied by a fit to a number of states in the known spectrum, which yields the 'best' values for the parameters. The values we obtain for the parameters of the Hamiltonian are shown in Table II.

TABLE II: Hamiltonian parameters obtained from the fit to a selection of known baryons.

m_σ (GeV)	m_s (GeV)	m_c (GeV)	m_b (GeV)	b (GeV^2)	$\alpha_{\rm Coul}$	$\alpha_{\rm con}$	$\alpha_{\rm SO}$ (GeV)	$\alpha_{\rm tens}$	C_{qqq} (GeV)
0.2848	0.5553	1.8182	5.2019	0.1540	≈ 0.0	1.0844	0.9321	-0.2230	-1.4204

We use three independent parameters for the strengths of the Coulomb, contact and tensor pieces of the potential. We believe that this is justified as the Hamiltonian we use is viewed as completely phenomenological. In many of the fits we have obtained, we find that the strength of the Coulomb interaction was consistently small, suggesting that, within this model, that interaction does not play a crucial role. We have also fixed the value of this coupling at 0.1 and 0.2 to investigate its effect on the other parameters and on the spectrum. When this is done, correlations among the parameters mean that they all change but no single parameter changes by more than a few percent. The spectrum also changes, with the masses of states shifting by up to 20 MeV, but with no significant degradation in the quality of the fit we obtain. Wave function size parameters also change by a few percent.

In contrast with the coupling constant for the Coulomb interaction, the coupling constant that results for the contact interaction is quite large, emphasizing the key role that this interaction plays in hadron spectroscopy. The tensor coupling also turns out to be relatively large, suggesting that this interaction, and the mixings that it induces between harmonic oscillator substates, are very important ingredients in the spectroscopy of baryons.

The string tension b is slightly smaller than the the nominal value of 0.18, but is larger than the value of 0.1425 GeV^2 recently obtained by Swanson and collaborators [24] in their treatment of charmonia. The quark masses we obtain are somewhat smaller than in our previous work, and this can be traced to two sources. First, in our previous work, we neglected the tensor interaction, and including it will necessarily result in modifications to all of the fit parameters. Second, we are not including the semileptonic decay rate of the Λ_c baryon in this fit. In Ref. [21] when this rate is included, it tended to push the quark masses, particularly those of the up, down and strange quarks, to higher values.

Perhaps the biggest influence in changing the values of the parameters has been the use of the SU(6) wave functions for the nucleons, Deltas and Omegas. In our previous work on the baryon spectra [21, 22], we used the so-called *uds* basis flavor wave functions. In this basis, the flavor wave functions of the proton and the Δ^+ are both *uud*, and the two states are constructed out of the same set of spin-space wave function components. For instance, for $J^P = \frac{1}{2}^+$, the full wave functions for both states would be written as *uud* multiplying the spin-space components of Eq. (12). Diagonalization would then yield the wave functions for seven states, five of which would be identified as nucleons, based on their symmetry under exchange of the first two quarks, while the remaining two would be identified as Deltas, as their wave functions would be fully symmetric under exchange of any pair of quarks. One important point to note is that the nucleons and Deltas were diagonalized together, so that they had the same wave function size parameters (α_ρ, α_λ), and the orthogonality of their wave functions arose from the coefficients of the spin-space components. One drawback of this basis was that it led to 'spurious' states in the case of Ω baryons, states that were the equivalent of nucleons, and these had to be identified and removed from the spectrum.

In the present work, we use the full SU(6) wave functions for nucleons and Deltas (as well as Omegas). This means that orthogonality of the wave functions arises from the flavor component. The wave functions for the five $J^P = \frac{1}{2}^+$ nucleons that arise in our model are shown in Eq. (15), while the two Deltas with the same spin and parity have wave functions

$$\Psi^{\Delta}_{1/2^+M} = \left(\eta_1^{\Delta}\left|{}^2 10(70,0^+)\frac{1}{2}^+\right\rangle + \eta_2^{\Delta}\left|{}^4 10(56',2^+)\frac{1}{2}^+\right\rangle\right), \tag{18}$$

with

$$\begin{aligned}\left|{}^2 10(70,0^+)\frac{1}{2}^+\right\rangle &= \phi^S\frac{1}{\sqrt{2}}\left(\chi^\rho\psi_{001000}(\rho,\lambda) + \chi^\lambda\psi_{000010}(\rho,\lambda)\right),\\ \left|{}^4 10(56',2^+)\frac{1}{2}^+\right\rangle &= \chi^S\phi^S\frac{1}{\sqrt{2}}\left(\psi_{2M_L0200}(\rho,\lambda) + \psi_{2M_L0002}(\rho,\lambda)\right).\end{aligned} \tag{19}$$

The sets of nucleons and Deltas are treated separately in the present work, so that nucleons with a particular spin and parity do not necessarily have the same wave function size parameters as Deltas with the same spin and parity.

It is instructive to compare the matrix element of one of the spin-dependent operators calculated in the *uds* and SU(6) bases, respectively, to demonstrate how the choice of basis plays a role in guiding the values of the parameters. Consider, for instance, the matrix element of $\mathbf{S}_1 \cdot \mathbf{S}_2$, evaluated using only the first component of Eq. (12), or the first component of Eq. (15). In either basis, this component is the predominant component of the wave function of the ground-state nucleon. In the *uds* basis, one finds

$$\langle \mathbf{S}_1 \cdot \mathbf{S}_2 \rangle = \left\langle \chi^\lambda\phi^\lambda \left| \mathbf{S}_1 \cdot \mathbf{S}_2 \right| \chi^\lambda\phi^\lambda \right\rangle = \frac{1}{4}, \tag{20}$$

while in the SU(6) basis, the result is

$$\langle \mathbf{S}_1 \cdot \mathbf{S}_2 \rangle = \frac{1}{2}\left\langle \left(\chi^\lambda\phi^\lambda + \chi^\rho\phi^\rho\right) \left| \mathbf{S}_1 \cdot \mathbf{S}_2 \right| \left(\chi^\lambda\phi^\lambda + \chi^\rho\phi^\rho\right) \right\rangle = \frac{1}{2}\left(\frac{1}{4}\left\langle\phi^\lambda|\phi^\lambda\right\rangle - \frac{3}{4}\left\langle\phi^\rho|\phi^\rho\right\rangle\right) = -\frac{1}{4}. \tag{21}$$

Since this particular matrix element plays a crucial role in the splitting between the ground state nucleon and Delta, for instance, it should not be a surprise that the fit parameters we obtain in this work differ significantly from those reported earlier.

Before discussing the results, we make one more comment about the parameters shown in Table II. One may argue that, in a model such as this, the parameters of the model can not be reported with an accuracy of more than two or three significant digits. However, we are attempting to fit a spectrum in which the masses of many states are known to better than one MeV, representing a precision of better than one part in several thousand in many cases. It is that precision that drives the precision to which our parameters need to be reported. As an example, the spectrum that we obtain with $m_u = 0.28$ GeV and $b = 0.15$ GeV^2 is significantly different from the spectrum we obtain using the values shown in the table.

A selection of states from the baryon spectrum we obtain is shown in Table III. For most states reported, the model provides a satisfactory description, with most masses being reproduced to within 20 MeV. The exceptions are the Σ_b and Σ_b^*, but the mass splitting between these two states is well reproduced. In comparison with this, the mass splittings among the states consisting of light quarks, such as the $\Delta - N$ or $\Sigma^* - \Sigma$ mass differences, are somewhat smaller than the experimental values. Nevertheless, these results give us some confidence that, when applied to hyperons with strangeness -1 and -2, the predictions will be reliable.

B. The Ξ States

Our model results for a portion of the Ξ spectrum is shown in Table IV. These results are discussed in some detail in the subsections below.

TABLE III: Baryon masses in GeV obtained in the quark model we use. The first two columns identify the state and its experimental mass, while the last column shows the masses that result from the model. If the experimental uncertainty in the mass of a state is less than 1 MeV, the uncertainty is not reported. Where appropriate, the masses shown are the average of different charge states. All masses are in GeV.

State	Experimental Mass	Model
$N(1/2^+)$	0.938	0.970
$\Delta(3/2^+)$	1.232 ± 0.001	1.232
$\Lambda(1/2^+)$	1.116	1.103
$\Sigma(1/2^+)$	1.189	1.210
$\Sigma(3/2^+)$	1.385	1.379
$\Lambda_c(1/2^+)$	2.285	2.268
$\Lambda_c(1/2^-)$	2.595	2.625
$\Lambda_c(3/2^-)$	2.628	2.636
$\Sigma_c(1/2^+)$	2.455	2.455
$\Sigma_c(3/2^+)$	2.518	2.519
$\Omega_c(1/2^+)$	2.698 ± 0.003	2.718
$\Lambda_b(1/2^+)$	5.624 ± 0.002	5.612
$\Sigma_b(1/2^+)$	5.812 ± 0.002	5.833
$\Sigma_b(3/2^+)$	5.833 ± 0.002	5.858

TABLE IV: The Ξ and Ω spectra obtained in this work. The first two columns identify the state and its experimental mass and uncertainty in the mass, while the last shows the masses that result from the model. The spins and parities shown in the first column are our quark model assignments. Masses are in GeV.

J^P	Ξ		Ω	
	Experiment	Model	Experiment	Model
$1/2^+$	$1.317\pm$ 0.001	1.325	-	2.175
	-	1.891	-	2.191
	-	2.014	-	-
$3/2^+$	$1.532\pm$ 0.001	1.520	1.672	1.656
	-	1.934	-	2.170
	-	2.020	-	2.182
$5/2^+$	$1.950\pm$ 0.015	1.936	-	2.178
	-	2.025	-	2.210
$7/2^+$	$2.025\pm$ 0.005	2.035	-	2.183
		2.148	-	-
$1/2^-$	$1.690\pm$ 0.010	1.725	-	1.923
	-	1.811	-	-
$3/2^-$	-	1.759	-	1.953
	$1.823\pm$ 0.005	1.826	-	-
$5/2^-$	-	1.883	-	-

1. *The Ξ States with Known J^P*

There are only three Ξ states with 'known' spin-parity: $\Xi(1317)$ with $J^P = \frac{1}{2}^+$, $\Xi(1530)$ with $J^P = \frac{3}{2}^+$ and $\Xi(1823)$ with $J^P = \frac{3}{2}^-$. Table IV shows that our model reproduces the masses of the $\frac{1}{2}^+$and $\frac{3}{2}^+$ states quite well. However, as with other states consisting solely of light quarks, the splitting between these states is

smaller than the experimental value. The model prediction for the mass of the lowest $\frac{3}{2}^-$ state is 1759 MeV, significantly smaller than the experimental value of 1823 MeV. The second $\frac{3}{2}^-$ is predicted to have a mass of 1826 MeV, very close to the mass of the experimental state. It would seem natural, therefore, to assign the experimental state to be the second $\frac{3}{2}^-$, and to suggest that the lowest $\frac{3}{2}^-$ is yet to be found. We make such an assignment with a cautionary note that models such as this often under predict the masses of negative parity states. Indeed, in this model, the $S_{11}(1535)$ is predicted about 90 MeV too light. On the other hand, the $\Lambda(1520)$ is relatively well described, with the model mass being 14 MeV greater than the experimental mass. For these states, inclusion of spin-orbit interaction that is more sensitive to the internal structure of the state is very important.

2. $\Xi(1690)$

In addition to the ground state and the state with $J^P = \frac{3}{2}^+$, the existence of the $\Xi(1690)$ is relatively certain, and its mass is fairly well known. Its spin and parity, however, have not yet been established. A recent study by the BaBar Collaboration [2, 6, 7] concludes that the spin is consistent with 1/2, while there have been suggestions that it is a negative parity state. In our model, we treat this state as having $J^P = \frac{1}{2}^-$, and the mass that results with this assumption is 1725 MeV, 35 MeV heavier than the nominal mass of the state. A more microscopic treatment of spin-orbit interactions, can be expected to drive this state to slightly lower mass. A number of other authors have predicted larger masses for this state [13, 14, 19, 20]. The level of accuracy we have obtained here is comparable to that obtained with some of the other baryon states we fit, so we claim that the spin-parity for the $\Xi(1690)$ state predicted by the model is $\frac{1}{2}^-$.

3. The 3-star Ξ *states*

The two other 3-star states known experimentally are the $\Xi(1950)$ and $\Xi(2030)$. The spin-parity is not known for either of these states, but the $\Xi(2030)$ state is reported as having $J^P \geq \frac{5}{2}^+$ [5]. There are a few model states that could be assigned to these two experimental states. In Table IV, we tentatively assign the $\Xi(1950)$ a spin-parity of $\frac{5}{2}^+$, in which case the model mass of 1936 MeV is in good agreement with the experimental mass. This state is also consistent with the model state with $J^P = \frac{3}{2}^+$ with a mass of 1934 MeV. Note that the PDG comments that there may be more than one baryon in this mass region, which would be completely consistent with our model predictions. For the $\Xi(2030)$, our model suggests that it has $J^P = \frac{7}{2}^+$, and the model mass of 2035 MeV is consistent with the experimental mass of 2025 MeV. The state is also consistent with the second $5/2^+$ state with a mass of 2025 MeV.

4. $\Xi(1620)$

The $\Xi(1620)$ is the only other Ξ state with a mass below 2 GeV. Experimental evidence for this state is very weak, with a number of searches yielding null results [25]. If this state exists, it poses a problem for the present model, as the lightest positive parity excited model state that lies near this mass would be a predominantly radial excitation with a mass more than 250 MeV larger than the nominal mass of this state. However, this kind of problem is one that has occurred in other sectors in models like this one, the most famous example of which is the Roper resonance with a mass of 1440 MeV: most quark models cannot produce a radial excitation that is as light as the Roper. Alternatively, this state could be assigned to be one of the predicted negative parity states, such as the lightest $3/2^-$ model state. In this case, the model state is about 130 MeV heavier than the experimental state. Whatever its parity, if the $\Xi(1620)$ exists, the present model, along with many others, would predict a mass that is too large by more than 100 MeV.

5. Ω Baryons

There is a single Ω that is known with any certainty. The lightest excited state known has a mass of 2252 MeV, but neither its spin nor its parity has been established. In our model, there are a number of states with masses near 2200 MeV. These include two states with $J^P = \frac{1}{2}^+$ (with predicted masses of 2175, and 2191 MeV), three excitations with $J^P = \frac{3}{2}^+$ (with predicted masses of 2170, 2182 and 2194 MeV), two $\frac{5}{2}^+$ states at 2178 and 2210 MeV, and a $\frac{7}{2}^+$ state at 2183 MeV. None of the masses of the negative parity states predicted in the model are sufficiently large for them to be considered as candidates for this experimental state. More experimental information is certainly needed in this sector of the baryon spectrum.

IV. CONCLUSION AND OUTLOOK

A nonrelativistic quark model was used to examine the 4 and 3-star known Ξ states and their possible spin-parity assignments. Recent experimental developments and measurements [2, 6, 8–10] have revived interest in these states, which can provide a window into the mechanism of confinement, and the relevant degrees of freedom in a baryon. Because the mass of the strange quark differs significantly from those of the up and down quarks, this window provides a somewhat different view from that provided by nonstrange baryons. The spin and parity of only a few of the well-known Ξ states are known, and based on the analysis in our model, tentative spin and parity assignments have been made for a number of excited cascades. We have found that the well-known states, $\Xi(1317)$, $\Xi(1530)$ and $\Xi(1823)$ are quite well reproduced by our model, while specific J^P assignments are made for the 3-star states with unknown spin-parity. Our model suggests that the $\Xi(1690)$ state should be assigned a J^P of $\frac{1}{2}^-$, while the $\Xi(1950)$ state is consistent with a model state of mass 1.934 GeV having $J^P = \frac{3}{2}^+$, as well as a model state of mass 1.936 GeV and $J^P = \frac{5}{2}^+$. The $\Xi(2030)$ is also consistent with two model states, one having $J^P = \frac{5}{2}^+$ (2.025 GeV), the other with $J^P = \frac{7}{2}^+$ (2.035 GeV). These predicted masses are in good agreement with the experimental ones. In the case of the Ω baryons, the sparse experimental data do not allow a meaningful comparison with the model.

Of course, an analysis of the masses alone is often not enough to make a definitive quark model assignment for the states like these. Analysis of the electromagnetic and strong couplings of these states to other cascades, as well as to hyperons containing a single strange quark, would go a long way in helping to pin down which model states correspond to which experimental states. Unfortunately, the data currently available do not allow any kind of meaningful quantitative analysis.

The work presented here can be extended in a number of directions. The heavy baryons, particularly the heavy cascades, are of particular interest, as activity at the B factories has been providing new information on these states. In addition, these states provide ideal testing grounds for ideas regarding diquark clustering in baryons and heavy quark symmetries. The spectrum of multiply-heavy baryons is also of interest, especially as the states found by the SELEX Collaboration have not been confirmed by other searches.

Acknowledgments

This work is supported by the Department of Energy, Office of Nuclear Physics, under contracts no. DE-AC02-06CH11357 (MP) and DE-AC05-06OR23177 (WR). WR is grateful to the Department of Physics, the College of Arts and Sciences and the Office of Research at Florida State University for partial support. The authors are grateful to V. Ziegler and J. Goity for useful discussions.

[1] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).

[2] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **97**, 112001 (2006).

[3] D. Teodoro *et al.*, Phys. Lett. **B77**, 451 (1978).

[4] S. F. Biagi *et al.*, Z. Phys. **C34**, 175 (1987).
[5] R. J. Hemingway *et al.*, Phys. Lett. **68B** 197 (1977).
[6] V. Ziegler, University of Iowa Thesis, 2007.
[7] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0607043.
[8] J. W. Price, J. Ducote, J. Goetz and B. M. K. Nefkens [CLAS Collaboration], Nucl. Phys. A **754**, 272 (2005).
[9] J. W. Price *et al.* [CLAS Collaboration], Phys. Rev. C **71**, 058201 (2005); L. Guo and D. P. Weygand [CLAS Collaboration], [arXiv:hep-ex/0601011].
[10] A. Alavi-Harati *et al.* [KTeV Collaboration], Phys. Rev. Lett. **95**, 081801 (2005).
[11] J. R. Batley *et al.* [NA48/I Collaboration], Phys. Lett. B **645**, 36 (2007)
[12] R. Witt, [arXiv:nucl-ex/0701063].
[13] K. T. Chao, N. Isgur and G. Karl, Phys. Rev. D **23**, 155 (1981).
[14] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).
[15] L. Y. Glozman and D. O. Riska, Phys. Rept. **268**, 263 (1996).
[16] L. Y. Glozman, W. Plessas, K. Varga and R. F. Wagenbrunn, Phys. Rev. D **58**, 094030 (1998).
[17] Y. Oh, Phys. Rev. D **75**, 074002 (2007).
[18] F. X. Lee and X. y. Liu, Phys. Rev. D **66**, 014014 (2002).
[19] R. Bijker, F. Iachello and A. Leviathan, Ann. Phys. **284**, 89 (2000).
[20] C. E. Carlson and C. D. Carone, Phys. Lett. **B484**, 260 (2000);
C. L. Schat, J. L. Goity and N. N. Scoccola, Phys. Rev. Lett. **88**, 102002 (2002);
J. L. Goity, C. Schat and N. N. Scoccola, Phys. Lett. B **564**, 83 (2003);
N. Matagne and F. Stancu, Phys. Rev. D **71**, 014010 (2005);
N. Matagne and F. Stancu, Phys. Rev. D **74**, 034014 (2006).
[21] M. Pervin, W. Roberts and S. Capstick, Phys. Rev. C **74**, 025205 (2006).
[22] M. Pervin, W. Roberts and S. Capstick, Phys. Rev. C **72**, 035201 (2005).
[23] N. Isgur and G. Karl, Phys. Rev. D **20**, 1191 (1979); Phys. Rev. D **19**, 2653 (1979), [Erratum-ibid. D **23**, 817 (1981)]; Phys. Rev. D **18**, 4187 (1978); Phys. Lett. B **72**, 109 (1977).
[24] T. Barnes, S. Godfrey and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
[25] J. K. Hassall *et al.*, Nucl. Phys. B **189**, 397 (1981);
S. R. Borenstein *et al.*, Phys. Rev. D **5**, 1559 (1972).